SPRING 2021: MATH 147 QUIZ 7 SOLUTIONS

In the questions below, you are asked to set up triple integrals. You do **not** need to calculate the resulting value. Each question is worth 5 points.

1. Using rectangular coordinates, set up $\int\int\int_B \sin(x) + \sin(y)\ dV$ as an iterated integral, where B is determined by the inequalities:

$$0 \leq x \leq \frac{\pi}{2}, \quad -\cos(x) \leq y \leq \cos(x), \quad -1 \leq z \leq 1.$$

Solution.

$$\int_0^{\frac{\pi}{2}} \int_{-\cos(x)}^{\cos(x)} \int_{-1}^{1} \sin(x) + \sin(z)\ dz\ dy\ dx.$$

2. Using rectangular coordinates, set up the triple integral that gives the volume of the solid bounded by $y^2 = x^2 + z^2$ and $y = a^2$, with $a > 0$.

Solution.

$$\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{\sqrt{x^2+y^2}}^{a^2} dy\ dz\ dx = \int_{-a}^{a} \int_{-\sqrt{a^2-z^2}}^{\sqrt{a^2-z^2}} \int_{\sqrt{x^2+y^2}}^{a^2} dy\ dx\ dz = \int_0^{a^2} \int_{-y}^{y} \int_{-\sqrt{y^2-x^2}}^{\sqrt{y^2-x^2}} dz\ dx\ dy.$$

3. Rewrite the integral $\int\int\int_B \frac{1}{x+3}\ dV$ using cylindrical coordinates, where B is determined by the inequalities:

$$0 \leq x^2 + y^2 \leq 9, \quad x, y \geq 0, \quad 0 \leq z \leq x + 3.$$

Solution.

$$\int_0^{\frac{\pi}{2}} \int_0^3 \int_0^{r\cos(\theta)+3} \frac{1}{r\cos(\theta) + 3} \cdot r\ dz\ dr\ d\theta.$$